Two-Parameter Gamma-Based SUH Derivation

P. R. Patil, S. K. Mishra, Nayan Sharma, and A. K. Swar

Abstract—A simple procedure is suggested to compute direct runoff hydrograph (DRH) using an improved two-parameter gamma distribution (2PGD) based synthetic unit hydrograph (SUH) and it is tested on several storm events recorded in 3 different hydrometeorological catchments. When compared, the computed hydrographs were more accurate than those due to popular Gray, SCS, and Snyder methods, because of avoidance of manual, subjective, and tiresome fitting of hydrograph through few data points for their adjustments for unit runoff volume. The improved 2PGD incorporates the available approximate, but accurate, empirical relations for estimation of β and λ factors governing the shape of the dimensionless unit hydrograph (UH) from the Nash parameter number of reservoirs (n). The SUH peak discharge per unit area per unit effective rainfall (q_p) and time to peak (t_p) have been derived from hydrologic and geomorphologic characteristics of the watershed, which is advantageous for field use. The objective of this paper is to propose a 2PGD-based method, test it on the data of 3 watersheds, and finally carry out a sensitivity analysis for model parameters. Model results during calibration and validation are very promising with average efficiency of each watershed exceeding 92% and 72%, respectively.

Index Terms—Direct runoff hydrograph, Gamma distribution, Synthetic unit hydrograph, Unit hydrograph.

I. INTRODUCTION

Routine measurements of runoff are often scanty, and therefore, unit hydrograph (UH) for ungauged catchments are derived using synthetic unit hydrograph (SUH) models dependent on catchment characteristics [18], [10]. The prominent approaches to synthesize UH are due to [18], [19], and [8]. These methods specify a few selected points on UH through which a curve is fitted by trial and error, which is subjective and tiresome for satisfying the unit volumetric condition. In these methods graphs or equations are provided to determine values of attributes such as peak flow rate, lag or rise time, base time, and hydrograph widths, W_{50} and W_{75} . These reasons coupled with the fact that a UH can reasonably be represented by a gamma distribution comprise the basis for its fitting.

References [11], [12], and [6] derived 2PGD from a cascade of *n*-linear reservoirs of equal storage coefficient K, known as instantaneous unit hydrograph (IUH) for a watershed. Since then, the gamma distribution is most commonly used in various forms depending on the discernible boundary conditions, such as peak and time to

peak. The improved 2PGD-based SUH method is easy to apply and meets the UH criterion of unity. The method can do away with the calculations for W_{50} and W_{75} . Reference [17] proposed an approximate analytical equation for calculating *n* and *K* (which define the shape and scale of the gamma distribution, respectively) from peak and time to peak of UH. Though the exact solution of *n* in terms of the non-dimensional shape factor $\beta (= q_p t_p)$ is difficult to evaluate, the Nash parameter *n* is accurately expressed mathematically in terms of β , by assuming *n* as a non-linear function of β , eliminating trials [2]. Shape factor can be taken as the form factor that quantifies the hydrograph peakedness and influences the hydrograph shape [17]. The discrete convolution allows the computation of DRH for a given rainfall-excess and UH derived from SUH.

II. MODEL DEVELOPMENT

A. 2PGD-Based SUH Method

For 2PGD-based SUH derivation, [7] proposed a theoretical expression for UH assuming Q to be proportional to $t^{x}e^{-yt}$ as follows:

$$Q = cAy(yt)^{x} e^{-yt} / \Gamma(x+1)$$
⁽¹⁾

where Q = discharge in ft³/s at time t, A = drainage area (mi²), x and y = parameters that can be represented in terms of peak discharge, and $\Gamma(x+1)$ = Gamma function of (x+1). With suitable change of variables and applying dimensional homogeneity, the following can be derived:

$$q = \frac{1}{k\Gamma n} \left(\frac{t}{k}\right)^{n-1} e^{\frac{-t}{k}}, k > 0, t > 0$$
⁽²⁾

$$\Gamma(n) = \int_{0}^{\infty} t^{n-1} - e^{(-t)}, t > 0$$
(3)

or

$$\Gamma(n) = (2\Pi)^{0.5} n^{n-0.5} e^{-n} \times \left(1 + \frac{1}{12n} + \frac{1}{288n^2} - \frac{139}{51840n^3} + \frac{571}{2488320n^4}\right)$$
(4)

where *n* and *K* are parameters that define the shape and scale of the gamma distribution; $\Gamma(n)$ is the gamma function of *n*, which equals (n-1)!; *e* is the base-number of Napierian logarithm; and *q* is the IUH (runoff depth resulting from effective rainfall in the form of Dirac delta-function, h⁻¹). The area under the curve defined by (2) is unity. Reference [4] related *n* and *K* as:

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$$K = t_P / (n-1) \tag{5}$$

Defining a non-dimensional parameter β as a product of q_p and t_p , (2) and (5) are combined into the following simpler form:

$$\beta = q_p t_p = (n-1)^{(n-1)} e^{-(n-1)} / \Gamma(n-1)$$
(6)

A simple numerical procedure i.e. Stirling's formula [1] was used by [2] to get an approximate solution of (6) as:

$$n = 5.53\beta^{1.75} + 1.04$$
 for $0.01 < \beta < 0.35$ (7a)

$$n = 6.29\beta^{1.998} + 1.157$$
 for $\beta \ge 0.35$ (7b)

Equations (7a) and (7b) were derived using numerical simulation and optimization, and can be used to estimate *n* for known values of q_p and t_p . These equations avoid the widely practiced trial-and-error solution of *n* for given β [17]. It is noted that β values less than 0.01 are seldom experienced in the field [17]. To obtain an SUH, the hydrograph parameters were related to catchment characteristics [13], [21].

B. Determination of Peak Discharge

A multitude of peak flow formulae related with catchment area, e.g., Dickens formula, are available in literature and these are of the form:

$$Q_P = C_d A^m \tag{8}$$

where Q_p = maximum flood peak (m³/s); A = catchment area (km²); and C_d , m = regression constants.

C. Determination of Time to Peak Discharge

Time to peak is estimated using the Snyder approach based on the concept of watershed lag from basin characteristics as:

$$t_P = C_t \left(L L_C \right)^{0.5} \tag{9}$$

where t_p = time to hydrograph peak (h), L = length of the main stream (km), L_c = length to watershed centroid from the outlet (km), and C_t is a regional constant representing variations in watershed slopes and storage characteristics.

D. Derivation of X-hr UH and DRH

The X-hr UH can be derived by averaging the known SUH ordinates at X-hr intervals as:

$$(X - hrUH)_t = \frac{1}{2} \left(SUH_t + SUH_{t-x} \right) \tag{10}$$

In discrete time domain,

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$$
(11)

where input is a series of 'M' pulses of constant rate. Equation (11) estimates direct runoff Q_n for given volume of m^{th} rainfall-excess pulse P_m and UH ordinates U_{n-m+1} .

E. Performance Evaluation

The performance of the proposed approach is evaluated using following criteria.

F. Nash-Sutcliffe Efficiency (NSE)

The NSE [14] measures the strength of correlation between two independent variables, and expressed as:

$$NSE = \left[1 - \left(\frac{RV}{IV}\right)\right] \times 100 \tag{12}$$

where
$$RV = \sum_{i=1}^{n} (Q_i - \hat{Q}_i)^2$$
 (13a)

$$IV = \sum_{i=1}^{n} \left(Q_i - \overline{Q_i} \right)^2$$
(13b)

Here, RV = remaining variance, IV = initial variance, Q_i = observed runoff for i^{th} pulse (m³/s), \hat{Q}_i = computed runoff for i^{th} pulse (m³/s), n = total number of observations, and \overline{Q}_i = overall mean runoff of storm event (m³/s). Efficiency varies at the scale of 0 to 100. It can also assume a negative value if RV > IV, implying that the variance in the observed and computed values is greater than the model variance. The efficiency of 100 implies a perfect fit between the observed and computed values.

G. Relative Error (RE)

The RE computes the deviation between the observed and simulated values with respect to the observed value as:

$$RE = \left[\left(X_{obs} - X_{comp} \right) / X_{obs} \right] \times 100 \tag{14}$$

Here, X_{obs} = observed value and X_{comp} = computed value. A high value of RE indicates greater deviation from the observed, and vice versa.

III. APPLICATION

A. Study Area and Data Used

Three watersheds were selected for application of proposed approach, (Fig. 1). The hydroclimatic and physiographic details of selected watersheds are given in Table I. Notations (C) and (V) indicate number of storm events used for calibration and validation, respectively.

The hydrograph parameters (i.e. q_p and t_p) were derived from physical catchment characteristics, viz., catchment area (A), and mainstream lengths (L and L_c) which are available from literature. Based on area, the watersheds can be categorized as to lie in the range from micro (0.0003 km², Cincinnati) to river basin (823.62 km², 3f sub-zone Godavari). 3f sub-zone Godavari has longest mainstream lengths, L = 61.08 km and $L_c = 22.54$ km as compared to Cincinnati, L = 0.024 km and $L_c = 0.014$ km. Jhandoo nala watershed is the steepest (slope = 50%) watershed while 3f sub-zone Godavari watershed is the mildest (slope = 0.12%) watershed.



Fig. 1. Study watersheds (a) Cincinnati (plan), (b) Jhandoo nala, and (c) 3f sub-zone Godavari.

TABLE I: HYDROCLIMATIC FEATURES AND PHYSIOGRAPHIC CHARACTERISTICS OF THE STUDY WATERSHEDS

| S. no (1) | Watershed/size/location (2) | Climate (3) | Avg. annual rainfall (mm) (4) | Soil (%) (5) | L km) (6) | L _C (km) (7) | Avg. Land slope (%) (8) | LULC (%) (9) | Source of watershed details/rainfall–runoff data (10) | No. of events used (11) |
|-----------------|--|--------------------------------|---|------------------------------|-----------------|-------------------------------|-------------------------------------|-------------------------|--|----------------------------------|
| 1 | Cincinnati (0.0003 km ²) Asphalt pavement at milestone 2.6 of I-75, Cincinnati, Ohio River, U.S.A. | Climatic Transition Zone | 1020 | Asphalt Pavement = 100 | 0.024 | 0.014 | 0.4 | UR=100 | [16] | C: 4 V: 3 |
| 2 | Jhandoo nala (0.177 km ²), Baldi River, Dehradun, Uttarakhand, India (32°23 ⁰ and 32°23½ ⁰ N) (78°7½ ⁰ and 78°8 ⁰ E) | Temperate | 2624 | SL=50 SICL=50 | 0.900 | 0.430 | 50 | OS=49 WL=47 AG=4 | [9] | C: 7 V: 7 |
| 3 | 3f sub-zone Godavari, (823.62 km ²), India (17° and 23° N) (76° and 83° E) | Humid | 1300 | S L CL | 61.08 | 22.54 | 0.124 | FO=50 AG=25 WL=25 | [3] [20] | C: 4 V: 4 |

Note: CL=Clay loam; SL=Sandy loam; L=Loam; S=Sand; SICL=Silty clay loam; LULC=Land use/Land cover; AG=Agriculture; FO=Forest; UR=Urban; OS=Orchard/Open scrub; WL=Waste land; C=Calibration; V=Validation.

IV. ANALYSIS AND DISCUSSION OF RESULTS

A. Determination of Rainfall-Excess and Direct Runoff

The effectiveness of IUH models depends on the conversion of rainfall volume to runoff volume, and therefore, it is necessary to separate the rainfall-excess hyetograph from the infiltration and observed DRH from the base flow [15]. The phi-index is determined by trial-and-error [5], such that

$$r_d = \sum_{m=1}^{M} \left(R_m - \Phi \Delta t \right) \tag{15}$$

where R_m = observed rainfall depth in mm or cm over the time interval m, r_d = direct runoff depth in mm or cm over the watershed, Φ = phi-index over the time interval length Δt (mm/hr or cm/hr), and M = number of non-zero pulses of rainfall-excess.

B. Development of SUHs

SUH derivation requires q_p (h⁻¹) and t_p (h) to be known apriory. Based on their determination, proposed procedure involves computation of q_p (h⁻¹) from maximum flood peak Q_p (m³/s) estimated using a regional Q_{P-A} relationship and t_p (h) using Snyder approach. This procedure has been calibrated using 15 of the total 29 randomly selected storm events, and validated on the remaining events. The notations (O) and (C) indicate observed and computed values of different variables, respectively. The procedure is explained, as an example, for event (4) of Jhandoo nala watershed, as follows:

For deriving a Gamma SUH (2) from known A, L, and L_c , the regional Q_P -A relationship (8) is used along with Snyder approach (9). Regional Q_{P} -A relationship estimates the maximum flood peak Q_p from known catchment area A =0.177 km² (Table I) with regression constants $C_d = 0.79$, and m = 0.75 (Table II). This estimated Q_p yields $q_p = 0.88$ h⁻¹ (Table II). Snyder approach uses $C_t = 0.33$ to estimate $t_p =$ 0.25 h (Table II) from known L = 0.900 km, and $L_c = 0.430$ km (Table I). The resulting parameters are K = 0.58 (5), $\beta =$ $q_p t_p = 0.22$ (6), and n = 1.43 (7a). The consequent NSE is quite high (= 98.05%), indicating excellent fit (Fig. 2). The relative errors in observed and computed runoff volumes, peaks, and time to peaks are 4.31%, 0.44%, and 0, respectively (Table II). The computed discharges in both rising and receding phases are slightly underestimated, leading to the volumetric overestimation at the start of receding phase and at tail end with gradual decrease in discharge.

Similarly, DRH for a storm event (4) of Cincinnati watershed is derived using $q_p = 4.37$ h⁻¹ and $t_p = 0.08$ h (Table II). The resulting efficiency (= 88.29%) indicates a satisfactory fit (Fig. 3). The relative errors in observed and computed runoff volumes, peaks, and time to peaks are 8.11%, 15.82%, and 11.11%, respectively, which are relatively high as compared to the previous event.

| | TABLE II: CALIBRATION RESULTS | | | | | | | | | | | | | | | |
|-------|-------------------------------|------|------|--------------------|------|----------------|--------|-------------------|-----------|----------|---------------------|---------|----------|------------------------|-------------|----------------------|
| Event | Δt (h) | Max. | C | q _p | C | t _n | Vol. | (m ³) | η | Peak | (m ³ /s) | Time to | Peak (h) | | RE (%) | |
| | | (mm) | Ċď | (h ⁻¹) | Ct | (h) | (0) | (C) | (%) | (0) | (C) | (0) | (C) | Vol. (m ³) | $Q_p(m3/s)$ | $t_{p}\left(h ight)$ |
| | | | | | | | | | Cincinn | ati | | | | | | |
| 1 | 0.02 | 1.67 | 2.31 | 37.86 | 0.17 | 0.02 | 9.64 | 9.08 | 93.98 | 5.E-03 | 6.E-03 | 0.62 | 0.63 | 5.88 | -20.71 | -2.70 |
| 2 | 0.02 | 0.87 | 1.41 | 44.37 | 0.15 | 0.01 | 2.77 | 2.59 | 95.74 | 4.E-03 | 4.E-03 | 0.75 | 0.75 | 6.80 | 0.31 | 0.00 |
| 3 | 0.03 | 0.39 | 0.12 | 8.18 | 0.51 | 0.05 | 0.12 | 0.11 | 93.95 | 2.E-04 | 3.E-04 | 0.23 | 0.20 | 4.12 | -9.77 | 14.29 |
| 4 | 0.08 | 0.14 | 0.02 | 4.37 | 0.91 | 0.08 | 0.13 | 0.12 | 88.29 | 1.E-04 | 1.E-04 | 0.75 | 0.67 | 8.11 | 15.82 | 11.11 |
| | Jhandoo nala | | | | | | | | | | | | | | | |
| 1 | 0.17 | 1.43 | 0.88 | 3.41 | 0.12 | 0.09 | 253 | 196 | 92.57 | 0.15 | 0.14 | 0.83 | 0.83 | 22.23 | 4.44 | 0.00 |
| 2 | 0.17 | 1.09 | 0.34 | 1.71 | 0.18 | 0.14 | 194 | 174 | 94.73 | 0.07 | 0.08 | 1.33 | 1.33 | 10.15 | -10.53 | 0.00 |
| 3 | 0.17 | 1.96 | 0.62 | 1.76 | 0.39 | 0.29 | 347 | 344 | 94.23 | 0.17 | 0.15 | 2.50 | 2.33 | 0.97 | 10.43 | 6.67 |
| 4 | 0.17 | 4.95 | 0.79 | 0.88 | 0.33 | 0.25 | 877 | 839 | 98.05 | 0.21 | 0.21 | 0.67 | 0.67 | 4.31 | 0.44 | 0.00 |
| 5 | 0.17 | 1.54 | 0.93 | 3.34 | 0.12 | 0.09 | 272 | 215 | 93.73 | 0.15 | 0.15 | 1.17 | 1.17 | 21.07 | -5.62 | 0.00 |
| 6 | 0.17 | 0.55 | 0.15 | 1.53 | 0.24 | 0.18 | 97 | 90 | 95.57 | 0.05 | 0.04 | 0.83 | 0.83 | 7.01 | 15.34 | 0.00 |
| 7 | 0.17 | 7.75 | 0.66 | 0.47 | 0.93 | 0.70 | 2214 | 2185 | 88.43 | 0.35 | 0.28 | 3.17 | 3.17 | 1.33 | 20.13 | 0.00 |
| | | | | | | | | 3f su | ib-zone G | iodavari | | | | | | |
| 1 | 1.00 | 4.31 | 0.86 | 0.13 | 0.35 | 3.10 | 8.E+06 | 8.E+06 | 95.43 | 250 | 249 | 8.00 | 9.00 | 0.70 | 0.20 | -12.50 |
| 2 | 1.00 | 5.25 | 1.02 | 0.13 | 0.52 | 4.53 | 7.E+06 | 7.E+06 | 97.94 | 255 | 247 | 7.00 | 8.00 | 0.31 | 3.19 | -14.29 |
| 3 | 1.00 | 1.17 | 0.15 | 0.09 | 0.68 | 5.97 | 3.E+06 | 3.E+06 | 86.10 | 65 | 64 | 11.00 | 12.00 | 1.18 | 2.26 | -9.09 |
| 4 | 1.00 | 2.77 | 0.40 | 0.10 | 0.49 | 4.32 | 5.E+06 | 5.E+06 | 90.44 | 140 | 125 | 11.00 | 12.00 | 5.61 | 10.65 | -9.09 |



Fig. 2. Calibration of a storm event (4) of Jhandoo nala watershed.



Fig. 3. Calibration of storm event (4) of Cincinnati watershed.

Calibration results of all the 15 events are shown in Table II. NSE varies from 86.10% (3f sub-zone Godavari) to 98.05% (Jhandoo nala). The relative errors in observed and computed runoff volumes, peaks, and time to peaks vary from 0.31% (3f sub-zone Godavari) to 22.23% (Jhandoo nala), 0.20% (3f sub-zone Godavari) to -20.71% (Cincinnati), and 0 (Cincinnati and Jhandoo nala) to \pm 14.29% (Cincinnati and 3f sub-zone Godavari), respectively. C_d is seen to vary from 0.02 (Cincinnati) to 2.31 (Cincinnati) whereas m has a constant value of 0.75. C_t varies from 0.12 to 0.93 for Jhandoo nala watersheds. The duration (Δt) of the developed UH/DRH varies from 0.02 h to 1 h, and the maximum rainfall excess from 0.14 (Cincinnati) to 7.75 mm (Jhandoo nala).

C. Model Validation

For validation, the pair/set of input parameters (i.e. q_p , t_p , C_d , m, and C_t) of calibrated events was averaged and used as an input for SUH derivation (Table III). DRH computed for storm event (2) of 3f sub-zone Godavari watershed from average input parameters are in close match with observed as in Fig. 4 with NSE (= 94.45%, Table IV). The relative error in runoff volumes is very low (= 0.40%) compared to relative errors in peaks (= -11.96%) and time to peaks (11.11%), respectively. Similarly, the model is validated on storm event (2) of Cincinnati watershed as in Fig. 5, but with low efficiency (= 72.01%, Table IV). The relative errors are 21.81% and -13.33%, respectively.



Fig. 4. Validation of storm event (2) of 3f sub-zone Godavari watershed.





TABLE III: AVERAGED CALIBRATED PARAMETERS USED IN VALIDATION

| S Ma | Watarahad | Averaged | | | | | | | |
|--------|----------------------|----------|------|---------------|-------|----------------------|--|--|--|
| 5. NO. | watersned | C_d | m | $q_p(h^{-1})$ | C_t | $t_{p}\left(h ight)$ | | | |
| 1 | Cincinnati | 0.97 | 0.75 | 23.70 | 0.43 | 0.04 | | | |
| 2 | 3f sub-zone Godavari | 0.61 | 0.75 | 0.11 | 0.51 | 4.48 | | | |
| 3 | Jhandoo nala | 0.62 | 0.75 | 1.87 | 0.33 | 0.25 | | | |

peaks, and time to peaks vary from 0.04% (Cincinnati) to -6.94% (Cincinnati), 4.96% (Jhandoo nala) to -61.97% (Jhandoo nala), and 0 (Cincinnati and Jhandoo nala) to -20% (Jhandoo nala), respectively.

D.Sensitivity Analysis

The validation results of all the 14 events are shown in Table IV. As seen, the time intervals (Δt) of the developed UH/DRHs deviate from 0.02 h to 1 h, and the maximum-rainfall excess from 0.01 (Cincinnati) to 8.11 mm (3f sub-zone Godavari). NSE varies from 63.13% (Cincinnati) to 95.58% (Jhandoo nala). Relative errors in runoff volumes,

Sensitivity analysis was performed to evaluate the impact of variation of q_p and t_p (-50% to +50% from their original value) on NSE (Table V). q_p is seen to be more sensitive for Cincinnati watershed whereas it is less sensitive for Jhandoo nala watershed. Similarly, t_p is more sensitive for 3f sub-zone Godavari watershed whereas it is less sensitive for Jhandoo nala watershed. Overall, NSE is more sensitive to q_p than t_p .

TABLE IV: VALIDATION RESULTS

| Event | Δt | Max. | | | qp | | t _p | Vol. | (m ³) | η | Peak | (m ³ /s) | Time to | Peak (h) | RE (%) | | |
|----------------------|------|------------|----------------|--------------------|------|------|----------------|--------|-------------------|--------|--------|---------------------|---------|------------------------|-------------|----------------------|--|
| | (h) | RE (mm) | C _d | (h ⁻¹) | Ct | (h) | (0) | (C) | (%) | (0) | (C) | (0) | (C) | Vol. (m ³) | $Q_p(m3/s)$ | $t_{p}\left(h ight)$ | |
| Cincinnati | | | | | | | | | | | | | | | | | |
| 1 | 0.02 | 1.81 | 0.97 | 23.70 | 0.43 | 0.04 | 3.88 | 3.88 | 82.34 | 7.E-03 | 7.E-03 | 0.32 | 0.35 | 0.04 | -13.37 | -10.53 | |
| 2 | 0.02 | 0.23 | 0.97 | 23.70 | 0.43 | 0.04 | 0.45 | 0.45 | 72.01 | 1.E-03 | 1.E-03 | 0.25 | 0.28 | 0.04 | 21.81 | -13.33 | |
| 3 | 0.03 | 0.01 | 0.97 | 23.70 | 0.43 | 0.04 | 0.03 | 0.03 | 63.13 | 3.E-05 | 2.E-05 | 0.20 | 0.20 | -6.94 | 23.12 | 0.00 | |
| Jhandoo nala | | | | | | | | | | | | | | | | | |
| 1 | 0.17 | 1.56 | 0.62 | 1.87 | 0.33 | 0.25 | 277 | 270 | 95.58 | 0.12 | 0.13 | 0.50 | 0.50 | 2.33 | -6.58 | 0.00 | |
| 2 | 0.17 | 0.66 | 0.62 | 1.87 | 0.33 | 0.25 | 116 | 113 | 87.58 | 0.06 | 0.06 | 1.17 | 1.17 | 2.50 | 4.96 | 0.00 | |
| 3 | 0.17 | 2.76 | 0.62 | 1.87 | 0.33 | 0.25 | 489 | 478 | 84.91 | 0.29 | 0.23 | 0.67 | 0.67 | 2.33 | 20.18 | 0.00 | |
| 4 | 0.17 | 0.26 | 0.62 | 1.87 | 0.33 | 0.25 | 46 | 45 | 92.32 | 0.03 | 0.02 | 2.83 | 2.83 | 2.33 | 11.88 | 0.00 | |
| 5 | 0.17 | 3.84 | 0.62 | 1.87 | 0.33 | 0.25 | 680 | 664 | 77.70 | 0.26 | 0.32 | 2.00 | 2.00 | 2.33 | -22.71 | 0.00 | |
| 6 | 0.17 | 2.98 | 0.62 | 1.87 | 0.33 | 0.25 | 528 | 516 | 44.86 | 0.16 | 0.25 | 2.67 | 2.50 | 2.33 | -61.97 | 6.25 | |
| 7 | 0.17 | 3.17 | 0.62 | 1.87 | 0.33 | 0.25 | 1122 | 1096 | 72.35 | 0.64 | 0.49 | 0.83 | 1.00 | 2.33 | 22.61 | -20.00 | |
| 3f sub-zone Godavari | | | | | | | | | | | | | | | | | |
| 1 | 1.00 | 3.22 | 0.61 | 0.11 | 0.51 | 4.48 | 5.E+06 | 5.E+06 | 67.04 | 330 | 156 | 8.00 | 7.00 | 0.86 | 52.85 | 12.50 | |
| 2 | 1.00 | 3.17 | 0.61 | 0.11 | 0.51 | 4.48 | 6.E+06 | 6.E+06 | 94.45 | 165 | 185 | 9.00 | 8.00 | 0.40 | -11.96 | 11.11 | |
| 3 | 1.00 | 3.67 | 0.61 | 0.11 | 0.51 | 4.48 | 7.E+06 | 7.E+06 | 92.64 | 205 | 178 | 10.00 | 11.00 | 0.40 | 13.39 | -10.00 | |
| 4 | 1.00 | 8.11 | 0.61 | 0.11 | 0.51 | 4.48 | 3.E+07 | 3.E+07 | 88.57 | 700 | 632 | 8.00 | 9.00 | 0.51 | 9.76 | -12.50 | |

| TABLE V: SENSITIVITY ANALYSIS OF Q_P and T_P | | | | | | | | | | | | | |
|--|----------------------|-------|--------------------|-------|---------------|-------|----------|-------|----------------------|-------|--------------------|-------|--|
| Watershed | Watershed Cincinnati | | | | | | o nala | | 3f sub-zone Godavari | | | | |
| % Variation | $q_p(h^{-1})$ | η (%) | t _p (h) | η (%) | $q_p(h^{-1})$ | η (%) | $t_p(h)$ | η (%) | $q_p(h^{-1})$ | η (%) | t _p (h) | η (%) | |
| -50 | 1.08 | 39.81 | 0.10 | 88.59 | 0.44 | 64.01 | 0.12 | 96.24 | 0.06 | 51.78 | 2.27 | 70.45 | |
| 0 | 2.17 | 94.48 | 0.19 | 94.48 | 0.88 | 98.05 | 0.25 | 98.05 | 0.13 | 97.94 | 4.53 | 97.94 | |
| 50 | 3.25 | 70.68 | 0.29 | 80.67 | 1.33 | 76.80 | 0.37 | 96.37 | 0.19 | 68.94 | 6.80 | 56.67 | |

V. CONCLUSION

The proposed approach is much simpler than the existing cumbersome trial-and-error solution for more accurate SUH derivation as proposed method can do away with the calculations for W50 and W75, and easily meet the UH criterion of unity. Furthermore, it enables determination of SUH for ungauged watersheds with little information on A, L, and L_c.

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